THERMAL FLUX DETERMINATION FROM TEMPERATURE MEASUREMENTS WITHIN A SENSOR

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An analytical solution of the fourth-order linear thermal conductivity equation is presented. This solution is used to develop a method for determining thermal flux on the surface of the body under study.

In recent years there has been much interest in determining thermal fluxes on the surfaces of bodies from measurements of temperatures within the body. The number of measurement points required may vary, depending on the method used for thermal flux determination and its accuracy. Thus, e.g., in [1, 2] a sensor with four thermocouples was considered, and thermal flux was calculated numerically during the course of heating.

However, according to [3], with such a number of measurement points one can obtain an analytic solution for the temperature field with quite high accuracy and then use Fourier's law to determine the unknown thermal flux. Such an approach simplifies calculations and increases accuracy. The present study will present one possible variant of this approach.

The sensors used in the thermal flux studies (Fig. 1) were in the form of copper cylinders with thermally insulated side surface to ensure uniform thermal flux in any arbitrary cylinder section. Four thermocouples were installed along the cylinder axis at the distances indicated in Fig. 1, and the emf from the thermocouples was measured by a K12-22 loop oscilloscope.

The thermal sensor was installed a certain distance from the nozzle of a plasmotron with axes of symmetry of sensor and nozzle coinciding. With the plasmotron in operation, a high-temperature gas flow acts on the endface of the sensor, producing a change in thermocouple emf, which is shown converted into degrees of temperature in Fig. 2. The data presented in Fig. 2 must now be used to determine the value of the thermal flux passing through the cylinder endface.

It was shown in [3] that solutions of third-order linear equations relative to an exact solution of the nonlinear equation have an error of the order of 5%, while for fourth-order equations the error is not more than 1%. Since we have four temperature measurements within the sensor, to determine the temperature field we must solve the following system of equations:

$$\frac{\partial^3\Theta}{\partial x^2\partial\tau} = a_0\frac{\partial^4\Theta}{\partial x^4} \quad (\tau > 0,\ R_1 < x < R_4), \tag{1}$$

$$\Theta|_{\tau=0} = 0 \quad (R_1 \leqslant x \leqslant R_4), \tag{2}$$

$$\Theta|_{x=R_1} = \varphi_1(\tau) \quad (\tau > 0), \tag{3}$$

$$\Theta|_{x=R_2} = \varphi_2(\tau) \quad (\tau > 0), \tag{4}$$

$$\Theta|_{x=R_3} = \varphi_3(\tau) \quad (\tau > 0), \tag{5}$$

$$\Theta|_{x=R_4} = \varphi_4(\tau) \quad (\tau > 0), \tag{6}$$

where $\Theta = t - t_0$; t_0 is the initial sensor temperature; a_0, thermal diffusivity of the sensor material at temperature t_0; and $\varphi_1(\tau)$, $\varphi_2(\tau)$, $\varphi_3(\tau)$, $\varphi_4(\tau)$, functions of the temperature change with time at points R_1, R_2, R_3, and R_4, respectively.

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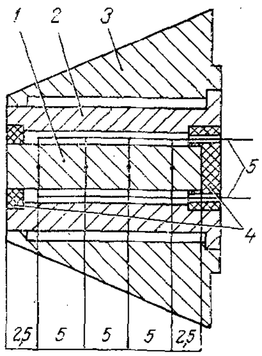

Fig. 1

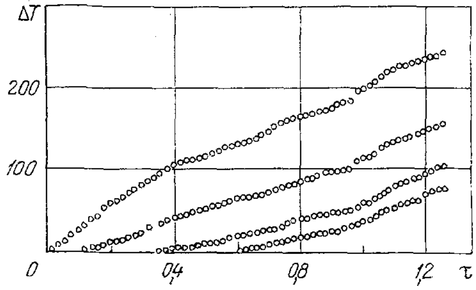

Fig. 2

Fig. 1. Diagram of thermal flux sensor: 1) copper cylinder; 2) body; 3) shield cone; 4) textolite inserts; 5) thermocouple leads.

Fig. 2. Change in temperature $\Delta T(^\circ K)$ with time τ (sec) at various thermal flux sensor sections.

Taking the Laplace transform of system (1)-(6) and retaining only first-order time derivatives of the functions $\varphi_1(\tau)$, $\varphi_2(\tau)$, $\varphi_3(\tau)$, and $\varphi_4(\tau)$ upon return to the original, we obtain the following solution of the original system:

$$\begin{aligned}\Theta(x,\tau) = {}& \varphi_1(\tau) + [\varphi_2(\tau) - \varphi_1(\tau)]\frac{x-R_1}{R_2-R_1}\frac{\psi_{11}(x)}{F_1} + [\varphi_3(\tau) - \varphi_1(\tau)]\frac{x-R_1}{R_3-R_1}\frac{\psi_{21}(x)}{F_1} + \\ & + [\varphi_4(\tau) - \varphi_1(\tau)]\frac{x-R_1}{R_4-R_1}\frac{\psi_{31}(x)}{F_1} + [\varphi_2'(\tau) - \varphi_1'(\tau)]\frac{x-R_1}{R_2-R_1}\frac{\psi_{11}(x)}{F_1}\left(\frac{\psi_{12}(x)}{\psi_{11}(x)} - \frac{F_2}{F_1}\right) + \\ & + [\varphi_3'(\tau) - \varphi_1'(\tau)]\frac{x-R_1}{R_3-R_1}\frac{\psi_{21}(x)}{F_1}\left(\frac{\psi_{22}(x)}{\psi_{21}(x)} - \frac{F_2}{F_1}\right) + [\varphi_4'(\tau) - \varphi_1'(\tau)]\frac{x-R_1}{R_4-R_1}\frac{\psi_{31}(x)}{F_1}\left(\frac{\psi_{32}(x)}{\psi_{31}(x)} - \frac{F_2}{F_1}\right),\end{aligned} \tag{7}$$

where

$$\begin{aligned}F_1 = {}& (R_4-R_2)[(R_4-R_2)^2+(R_2-R_1)^2+(R_4-R_1)^2] - (R_4-R_3)[(R_4-R_3)^2+(R_4-R_1)^2+(R_3-R_1)^2] - \\ & - (R_3-R_2)[(R_3-R_2)^2+(R_3-R_1)^2+(R_2-R_1)^2];\end{aligned}$$

$$\begin{aligned}F_2 = {}& (R_4-R_2)\frac{(R_2-R_1)^2(R_4-R_2)^2-(R_4-R_1)^2(R_4-R_2)^2+(R_4-R_1)^2(R_2-R_1)^2}{24a_0} - \\ & - (R_4-R_3)\frac{(R_4-R_1)^2(R_4-R_3)^2+(R_3-R_1)^2(R_4-R_3)^2+(R_3-R_1)^2(R_4-R_1)^2}{24a_0} - \\ & - (R_3-R_2)\frac{(R_3-R_1)^2(R_3-R_2)^2+(R_2-R_1)^2(R_3-R_2)^2+(R_2-R_1)^2(R_3-R_1)^2}{24a_0} + \\ & + (R_4-R_2)\frac{(R_4-R_2)^4+(R_2-R_1)^4+(R_4-R_1)^4}{80a_0} - (R_4-R_3)\frac{(R_4-R_3)^4+(R_4-R_1)^4+(R_3-R_1)^4}{80a_0} - \\ & - (R_3-R_2)\frac{(R_3-R_2)^4+(R_3-R_1)^4+(R_2-R_1)^4}{80a_0},\end{aligned}$$

and the functions $\psi_{11}(x)$, $\psi_{21}(x)$, $\psi_{31}(x)$ and $\psi_{12}(x)$, $\psi_{22}(x)$, $\psi_{32}(x)$ are obtained from F_1 and F_2, respectively, by sequential substitution of R_2, R_3, R_4 for x.

Solution (7) is real within the region $R_1 \leqslant x \leqslant R_4$. However, in view of the small size of the interval $[0, R_1]$, solution (7) can be extended to the interval $0 \leqslant x \leqslant R_4$. Then in accordance with Fourier's law we have the following expression for definition of the thermal flux at x = 0:

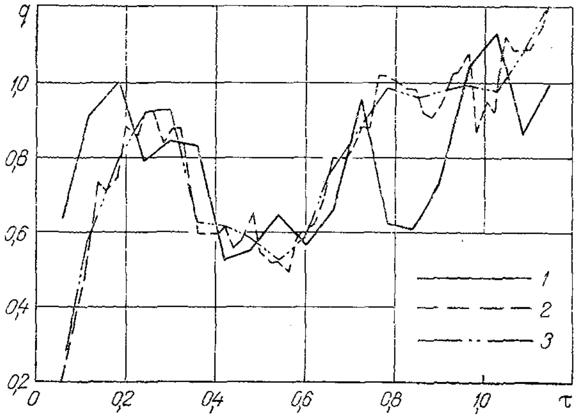

Fig. 3. Thermal flux q (10^7 W · m^{-2}) vs time τ (sec) as calculated by: 1) method of [1], time step 0.06 sec; 2) present method, $\Delta\tau$ = 0.02 sec; 3) $\Delta\tau$ = 0.06 sec.

$$q(\tau) = -[\lambda_0 + \lambda_1 \Theta(x, \tau)] \left. \frac{\partial \Theta}{\partial x} \right|_{x=0}, \quad (8)$$

where λ_0 and λ_1 are the corresponding coefficients of the temperature dependence of the thermal conductivity coefficient. The value of the second term in brackets in Eq. (8) may comprise 15% of λ_0.

Figure 3 shows results of thermal flux calculations for various time intervals. For comparison, data from a calculation by the method of [1] are also shown.

It follows from Fig. 3 that the method proposed here provides satisfactory agreement with the data of other authors with minimum calculation effort.

NOTATION

t, temperature; τ, time; a_0, thermal diffusivity; x, coordinate along sensor axis; R_1, R_2, R_3, R_4, coordinates of thermocouple positions; q, thermal flux density; λ_0, λ_1, coefficients in temperature dependence of thermal conductivity.

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